SOLUTION OF THE THERMAL PROBLEM OF THE VAPORIZATION OF CONICAL BODIES IN STRONG RADIATION FLUXES

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The paper gives approximate calculations for the rate of change of the axial dimensions and the temperature at the vertex of conical bodies vaporizing under the action of a powerful laser radiation.

Lykov [1, 2] discussed the possibility of using hyperbolic equations with discontinuous coefficients in the formulation of heat-conduction problems with phase transformations. As the equation which, near the phase-transition front, simultaneously describes the heat balance both within each phase and on the surface between the phases, we propose to use the following:

$$\frac{L\rho k_i}{\left|\left[k_1\frac{\partial T_1}{\partial t}-k_2\frac{\partial T_2}{\partial t}\right]_{f=0}\right|}\frac{\partial^2 T}{\partial t^2}+C_i\rho\frac{\partial T_i}{\partial t}=\operatorname{div}(k_i\operatorname{grad}T_i)\ (i=1,\ 2). \tag{1}$$

We shall show that Eq. (1) implicitly contains the Stefan condition

$$\left[k_1\frac{\partial T_1}{\partial n}-k_2\frac{\partial T_2}{\partial n}\right]_{f=0}=\pm L\rho\frac{\left|\dfrac{\partial f}{\partial t}\right|}{H_f}. \tag{2}$$

We shall consider the field of isotherms in a narrow region near the front. The temperature at each isothermal surface $f(x, y, z, t) = \text{const}$ is given by means of a function of $f(x, y, z, t)$, i.e.,

$$T=T(f(x,\ y,\ z,\ t)). \tag{3}$$

We transform Eq. (1) to the variable f:

$$\frac{L\rho k_i}{\left|\left[k_1\frac{\partial T_1}{\partial t}-k_2\frac{\partial T_2}{\partial t}\right]_{f=0}\right|}\left[\frac{d^2T_i}{df^2}\left(\frac{\partial f}{\partial t}\right)^2+\frac{dT_i}{df}\frac{\partial^2 f}{\partial t^2}\right]+C_i\rho\frac{dT_i}{df}\frac{\partial f}{\partial t}=k_i\left(\frac{d^2T_i}{df^2}H_f^2+\frac{dT_i}{df}\Delta f\right). \tag{4}$$

We integrate both sides of Eq. (4) with respect to f and then perform the operation of finding the jump at the passage through the frontal surface $f = 0$. For the sake of definiteness, we shall assume that the phase-transition front corresponds to the zero temperature $T(f)_{f=0} = 0$. We obtain

$$\frac{L\rho\left[k_1\dfrac{dT_1}{df}-k_2\dfrac{dT_2}{df}\right]_{f=0}}{\left|\left[k_1\dfrac{dT_1}{df}-k_2\dfrac{dT_2}{df}\right]_{f=0}\right|\left|\dfrac{\partial f}{\partial t}\right|}\left(\frac{\partial f}{\partial t}\right)^2=\left[k_1\frac{dT_1}{df}-k_2\frac{dT_2}{df}\right]_{f=0}H_f^2. \tag{5}$$

We perform the cancellations in formula (5) and make use of the relation for the transformation of the derivative

$$\frac{\partial T}{\partial n}=\frac{dT}{df}\frac{\partial f}{\partial n}=\frac{dT}{df}H_f, \tag{6}$$

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TABLE 1. Rate of Change of the Axial Dimensions and the Temperature at the Vertex of the Body as Functions of the Density of the Incident Flux (material:aluminum)

q, W/m²	g, m/sec	T_0, K	p, $\frac{1}{\mathrm{m}}\cdot 10^3$
10^{10}	0,37	3965	0,13
$5\cdot 10^{10}$	1,88	4890	0,66
$1{,}1\cdot 10^{11}$	4,18	5525	1,47
$5\cdot 10^{11}$	19,7	7370	6,86

after which Eq. (5) becomes

$$\pm L\rho\left|\frac{\partial f}{\partial t}\right| = \left[k_1\frac{\partial T_1}{\partial n} - k_2\frac{\partial T_2}{\partial n}\right]_{f=0} H_f\,. \tag{7}$$

It is not difficult to see that (7) is equivalent to the Stefan condition (2).

Kislitsyn and Morar [3] considered the problem of the vaporization of conical bodies in strong radiation fluxes. The solution of this problem was obtained for the following simplifying assumptions. It was assumed that the vaporization takes place in a vacuum under the action of a flux directed along the axis of symmetry of the body and having density q. Condensation was disregarded. The authors investigated the stationary behavior of the process, when the rate of change of the axial dimensions, g, was constant and the temperature field near the interface between the phases depended on the coordinates and time only in terms of the frontal variable f(x, y, z, t), i.e.,

$$T = T(f(x,\ y,\ z,\ t))\,. \tag{8}$$

A simultaneous analysis of the heat-conduction equation

$$C\rho\,\frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\,\frac{\partial T}{\partial t} + \frac{\partial^2 T}{\partial z^2}\right] \tag{9}$$

and the heat-balance condition

$$-k\,\frac{\partial T}{\partial n} = \frac{q - L\rho g}{H_f} \tag{10}$$

showed that these equations did not admit of a coincident frontal solution. While Eq. (9), for the condition $\frac{d^2T}{df^2}\Big/\frac{dT}{df} = p$, yielded for the frontal surface, the expression

$$z = \frac{1}{p}\ln\left|I_0\left(\sqrt{p\left(p - \frac{g}{a}\right)}\,r\right)\right|, \tag{11}$$

Eq. (10), after transformation by means of the relation (8) to the variable f, leads to

$$z = Ar + A_0 \quad (A \text{ and } A_0 - \text{const})\,. \tag{12}$$

The resulting discrepancy can be eliminated if we use Eq. (1), which describes the heat balance both within each phase and at the passage through the interface between the phases.

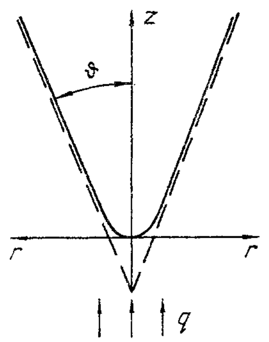
Suppose that a flux with density q impinges on a body which has a conical construction (Fig. 1). Taking account of the assumptions made, we analyze the simultaneous processes of heat conduction and formation of matter in the new phase in a narrow region near the phase-transition surface. We shall carry out this analysis on the basis of Eq. (1) and the kinetic condition

$$g = v_0\exp[-\alpha/T]\,. \tag{13}$$

We transform Eq. (1) to the variable f:

$$f = N(r) + gt - z\,, \tag{14}$$

where the function N(r), determining the configuration of the vaporization surface, remains to be defined. We have



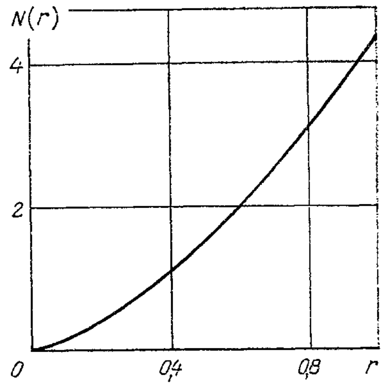


Fig. 1 Fig. 2

Fig. 1. Profile of conical body.

Fig. 2. Configuration of the vaporization front for $\theta = 10°$, $q = 1.1 \cdot 10^{11}$ W/m². N(r), 10^{-3} m; r, 10^{-3} m.

$$\gamma g \frac{d^2T}{df^2} + g\frac{dT}{df} = a_2\left(\frac{d^2T}{df^2}(1+(N'))\right) + \frac{dT}{df}\left(N'' + \frac{1}{r}N'\right), \tag{15}$$

where

$$\gamma = \frac{L\rho k_2}{\left|\left[k_1\frac{dT_1}{df} - k_2\frac{dT_2}{df}\right]_{f=0}\right| C_2\rho}. \tag{16}$$

The assumption that the process is stationary enables us to suppose that near the isotherm corresponding to the phase-transition surface the temperature field can be approximately expressed by means of the exponential function

$$T = T_0 \exp[pf], \tag{17}$$

where T_0 is the temperature at the vertex of the body. From this it follows that

$$\frac{d^2T}{df^2} \Big/ \frac{dT}{df} = p \tag{18}$$

and expression (15) can be transformed to the form

$$p\gamma\frac{g}{a_2} + \frac{g}{a_2} = p(1+(N')^2) + N'' + \frac{1}{r}N'. \tag{19}$$

Since the phase-transition front is an isotherm, we can use the equation for the propagation of isotherms which was obtained in [4] and find an expression for γ:

$$\frac{\gamma}{g} = \frac{a_2}{g^2}, \quad \text{or} \quad \gamma = \frac{a_2}{g}. \tag{20}$$

Substituting γ from (20) into (19) and making the necessary transformations, we obtain an equation for the unknown function N(r):

$$N'' + \frac{1}{r}N' + p(N')^2 - \frac{g}{a_2} = 0. \tag{21}$$

As in [3], we make use of the fact that the obvious conditions

$$N(0) = 0, \quad N'(0) = 0, \quad \lim_{r\to\infty}|N'(r)| = \operatorname{ctg}\theta, \quad \lim_{r\to\infty}|N''(r)| = 0 \tag{22}$$

are satisfied, and we write the solution of Eq. (21) in the form

TABLE 2. Rate of Change of Axial Dimensions and Temperature at the Vertex of the Body as a Function of the Angle θ

ϑ, deg	g, $\frac{\text{m}}{\text{sec}}$	T_0, K	p, $\frac{1}{\text{m}}\cdot 10^3$
2	4,24	5538	0,05
5	4,23	5535	0,37
8	4,21	5530	0,94
10	4,18	5525	1,47
20	3,97	5479	5,99
30	3,64	5403	13,79

$$N(r) = \frac{1}{p} \ln \left[I_0 \left(\sqrt{p \frac{g}{a_2} r} \right) \right], \tag{23}$$

(23)

where

$$p = \frac{g}{a_2} \operatorname{tg}^2 \theta. \tag{24}$$

Making use of the relations (13), (16), and (20), we find an equation for determining g;

$$\frac{g}{a_2} = \frac{(q - L\rho g) \ln (v_0/g) \operatorname{ctg}^2\theta}{\alpha k_2}. \tag{25}$$

Since Eqs. (1) and (13) were analyzed simultaneously only in a narrow region near the front, the calculations performed on the basis of formula (25) will have sufficient accuracy in the case when the problem satisfies the condition

$$\operatorname{tg}^2 \theta \leqslant 1. \tag{26}$$

The quantity a_2/g characterizes the dimensions of the region adjacent to the front, where there is a substantial temperature gradient; this is the so-called "warmed" layer. The condition (26) means that the thickness of the warmed zone should not exceed the characteristic dimensions of the body. On the basis of (13) and (25), we performed numerical calculations of the constants of vaporization of conical aluminum bodies for various values of the half-apex angle approximating the body of the cone and the density of the external incident flux (Tables 1 and 2). The values of the thermophysical constants for aluminum were taken from [4]. Figure 2 shows a graph of function N(r), which determines the configuration of the stationary front of the vaporization of a conical aluminum body with $\theta = 10°$ and $q = 1.1 \cdot 10^{11}$ W/m^2.

It should be noted that the values for the rate of change of the axial dimensions of conical bodies should not differ greatly from the values for the rate of deepening of a hole in metal when the material vaporizes under the action of a laser beam. In [6] an exact numerical solution is given for the problem of hole formation. A comparison of the values g of the rate of change of the axial dimensions from Tables 1 and 2 and the analogous values from [3] with the values of the rate of deepening of a hole which are given in [6] shows that the calculations performed here have served to refine the calculations of [3].

NOTATION

T, T_i, temperature (the subscript i indicates phase; i = 1, 2); k, thermal conductivity; x, y, z, spatial coordinates in a Cartesian system; r, radial coordinate in a cylindrical coordinate system; t, time coordinate; L, specific heat of the phase transition; q, density of the incident external heat flux; C, specific heat capacity; ρ, density; f(x, y, z, t), equation of the frontal surface; g, rate of change of the axial dimensions of conical bodies; ϑ, half-apex angle of cone approximating the body; T_0, temperature at the vertex of the body; v_0, preexponential multiplier in the kinetic equation; α, multiplier in the exponent in the kinetic equation; $I_0(z)$, modified zero-order Bessel function; $H_f^2 = (\partial f/\partial x)^2 + (\partial f/\partial y)^2 + (\partial f/\partial z)^2$, first differential parameter; $\Delta f = \partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2$, Laplace operator.

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